

7.4 #9

Integrate

$$\int \frac{x+1}{6x+x^2} dx$$

$$6x+x^2 = x(6+x)$$

$$\Rightarrow \frac{x+1}{6x+x^2} = \frac{A}{x} + \frac{B}{6+x}$$

$$\Rightarrow A(6+x) + Bx = x+1$$

$$A6 + Ax + Bx = x+1$$

$$(A+B)x + 6A = x+1$$

then

$$A+B = 1 \quad 6A = 1$$

$$B = 1-A \quad A = 1/6$$

$$B = 5/6$$

$$\int \frac{x+1}{6x+x^2} dx$$

$$= \int \left(\frac{1/6}{x} + \frac{5/6}{6+x} \right) dx$$

$$= \frac{1}{6} \int \frac{1}{x} dx + \frac{5}{6} \int \frac{1}{6+x} dx$$

$$= \frac{1}{6} \ln|x| + \frac{5}{6} \ln|6+x| + C$$

$$= \frac{1}{6} (\ln|x| + 5 \ln|6+x|) + C$$